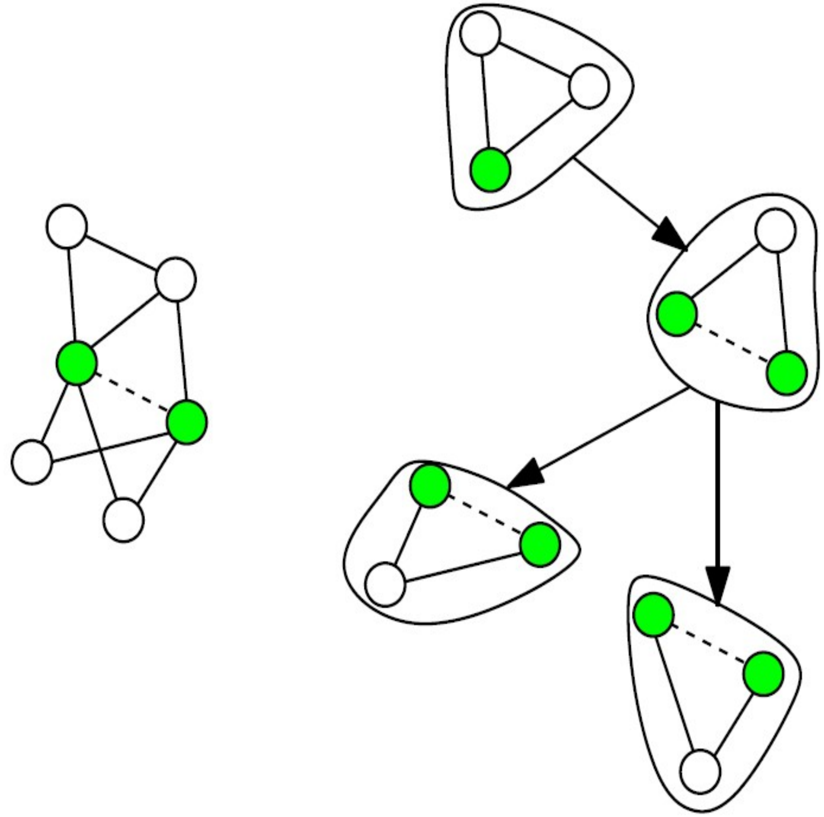
Minimum Cycle Bases in Partial 2-Trees



Carola Doerr G. Ramakrishna Jens M. Schmidt

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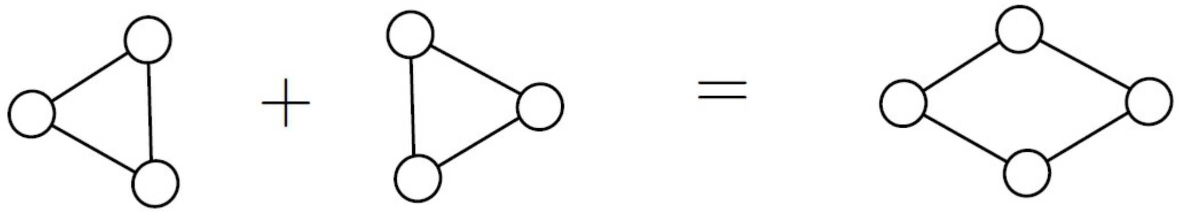

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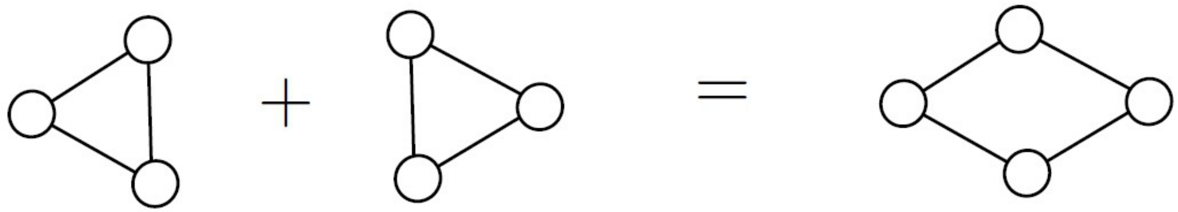

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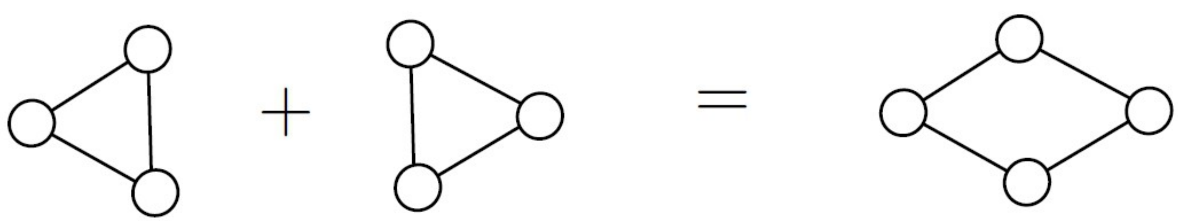
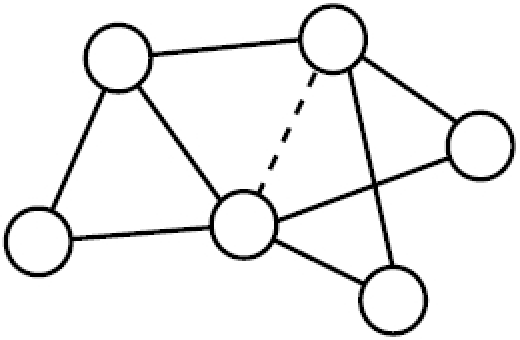


- minimum cycle basis = minimize sum of edge-weights

Partial 2-Trees

- 2-trees:
  - Start with a triangle
  - Add iteratively new vertex that is adjacent to a K_2



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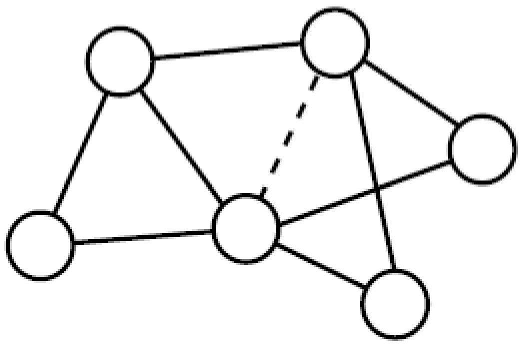

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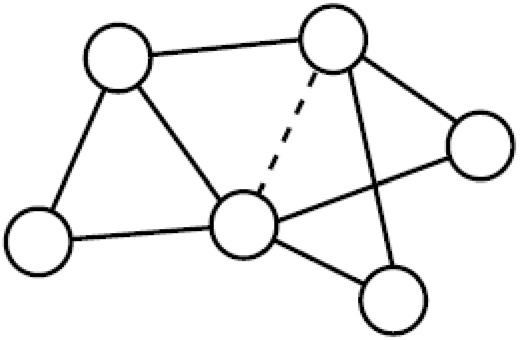

- partial 2-trees = subgraphs of 2-trees
- outerplanar graphs ⊂ partial 2-trees

Recent Work

- For general graphs:
  - Deterministic: $O(m^2 n / \log n)$ [Amaldi-Iuliano-Rizzi '10]
  - Monte-Carlo: $O(m^\omega)$ [“-Jurkiewicz-Mehlhorn '09]

Recent Work

	[LL 2010]	[Our Result]	[BSW 2010]
	Outerplanar	Partial 2-trees	Planar
Preprocessing time	$O(n)$	$O(n)$	$O(n \log^5 n)$
Space required	$O(n)$	$O(n)$	$O(n \log n)$
Reporting time	$size(MCB)$	$size(MCB)$	$size(MCB)$

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Can be big: $\Omega(n^2)$

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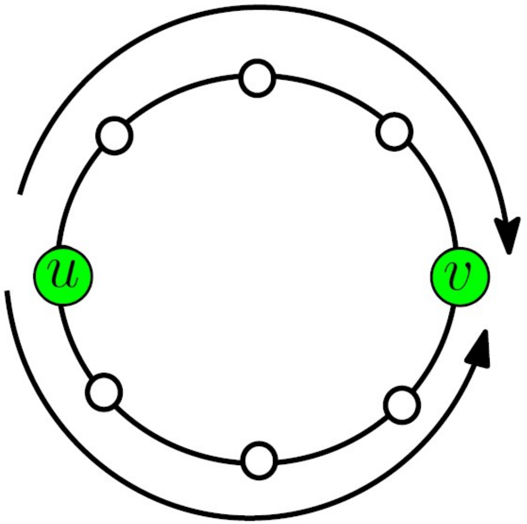

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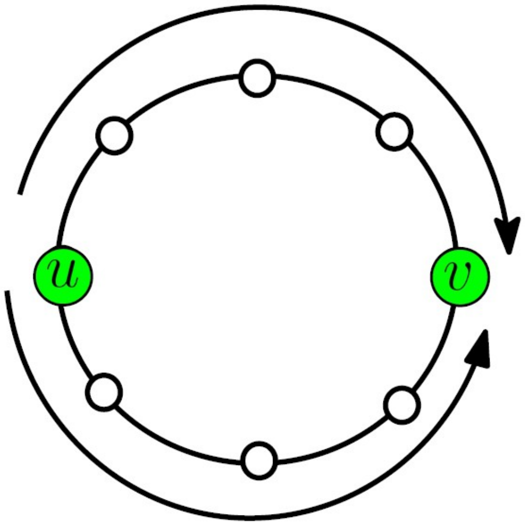

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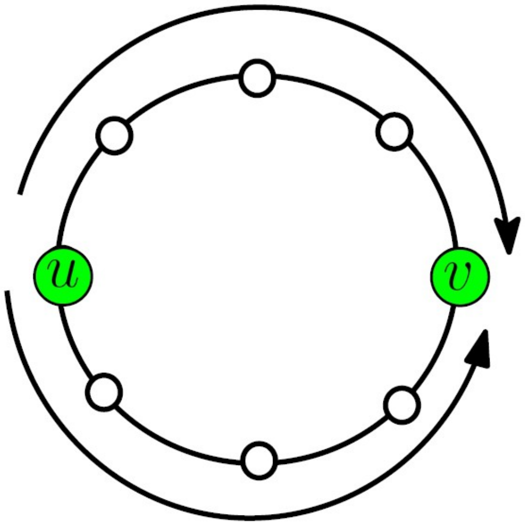

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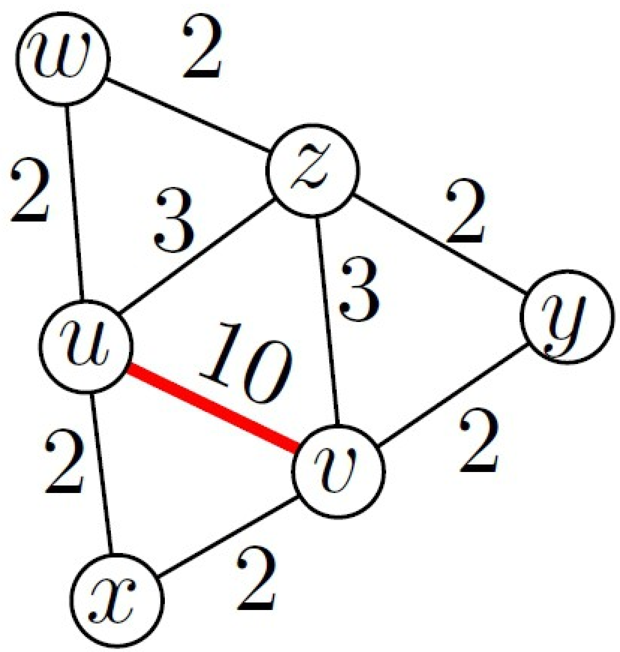
Thm [N.+Ramakrishna '12]: MCB = SC for partial 2-trees

Tight Edges

- Edge (u,v) tight if sp(u,v)=(u,v), otherwise loose
- Every loose edge appears in exactly one SC



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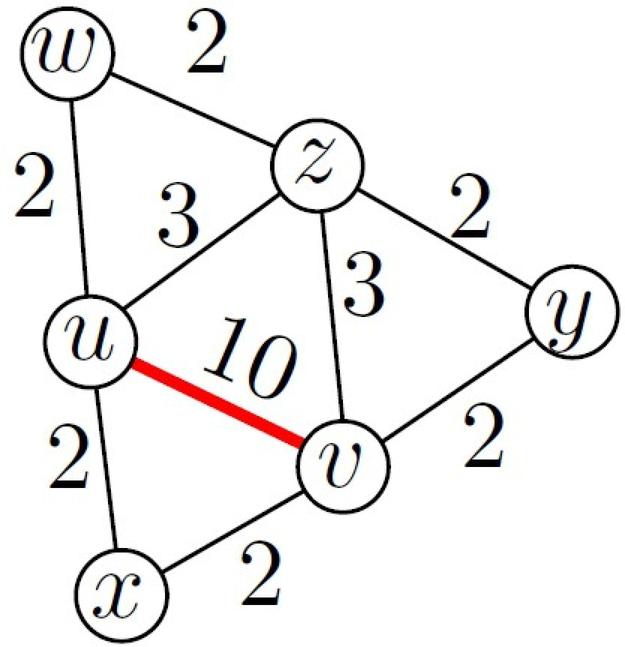
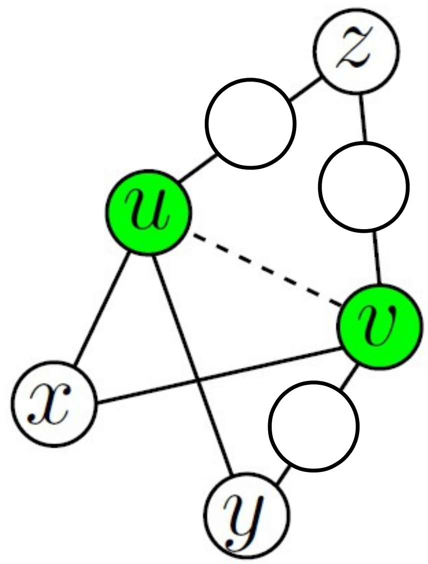


- Precompute these cycles => G contains only tight edges

Decomposition

- If partial 2-tree has no $K_{2,3}$-subdivision => outerplanar!



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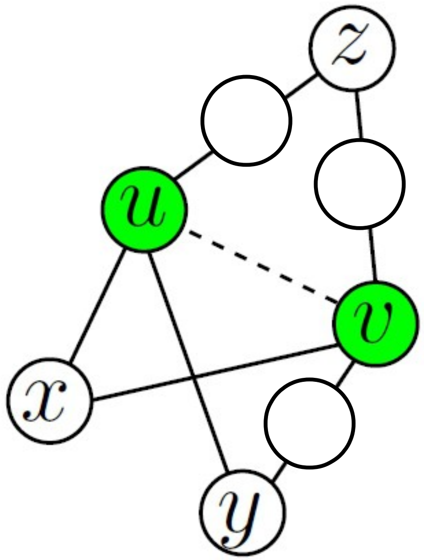

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Thm [N.+Ramakrishna '12]: $SC(G) = SC(G_1) \cup \ldots \cup SC(G_r)$

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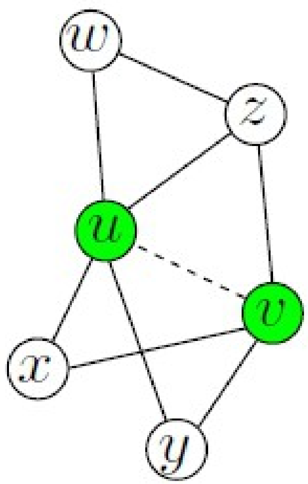

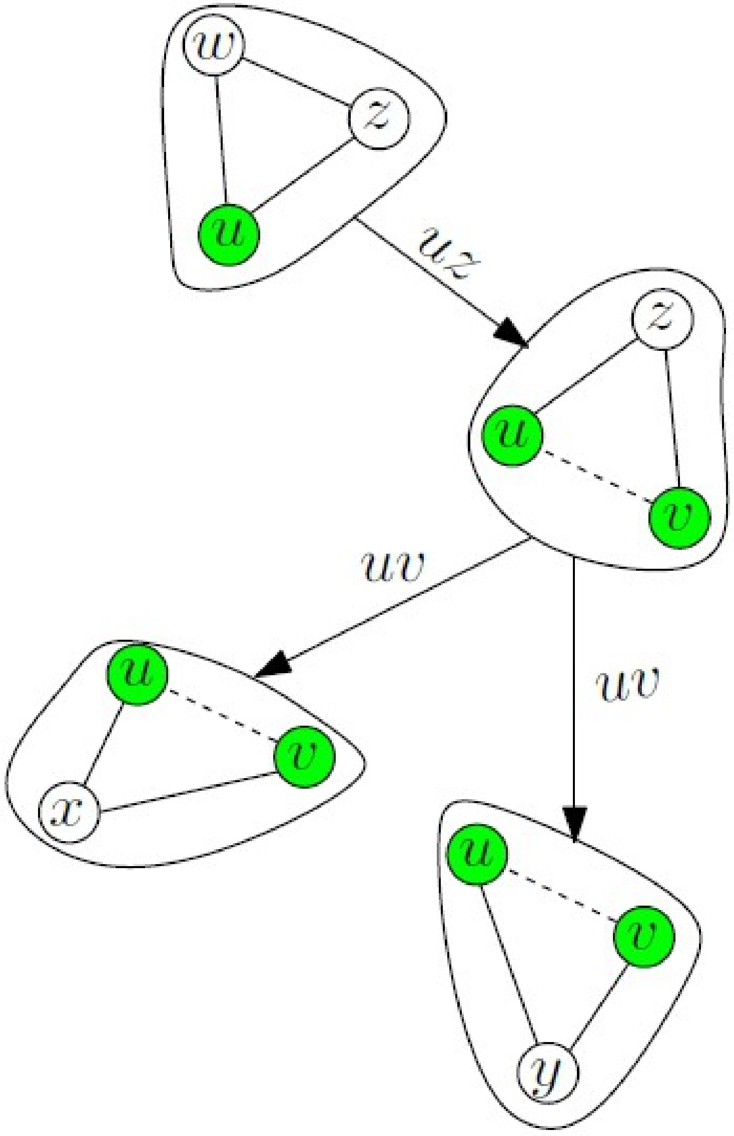

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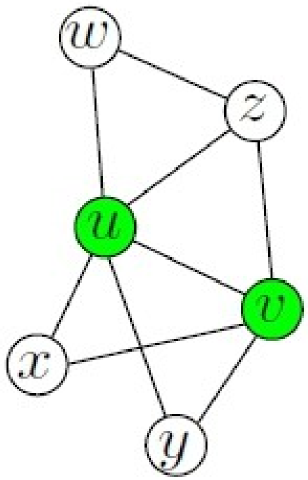

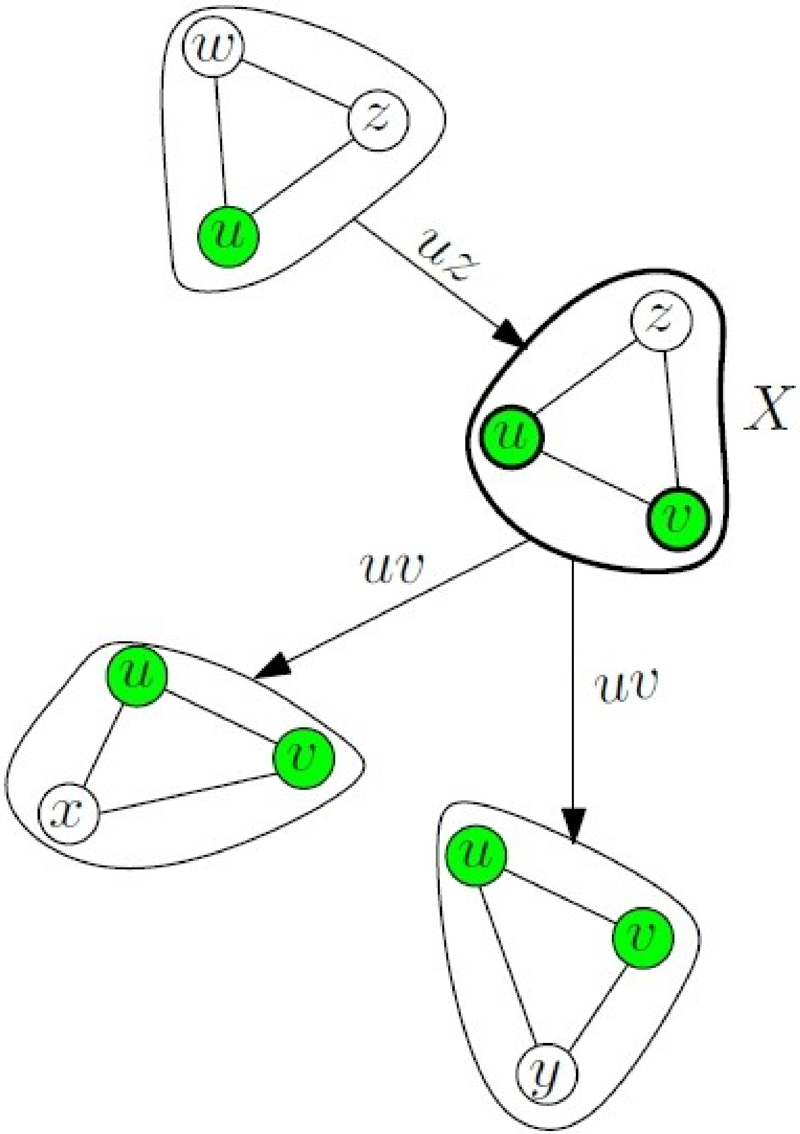

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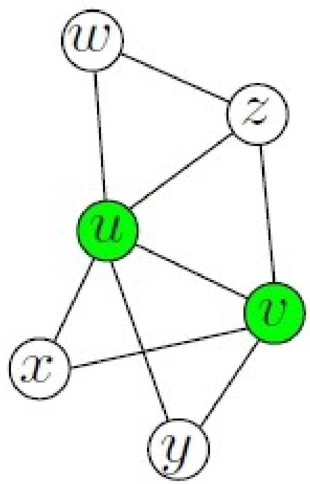

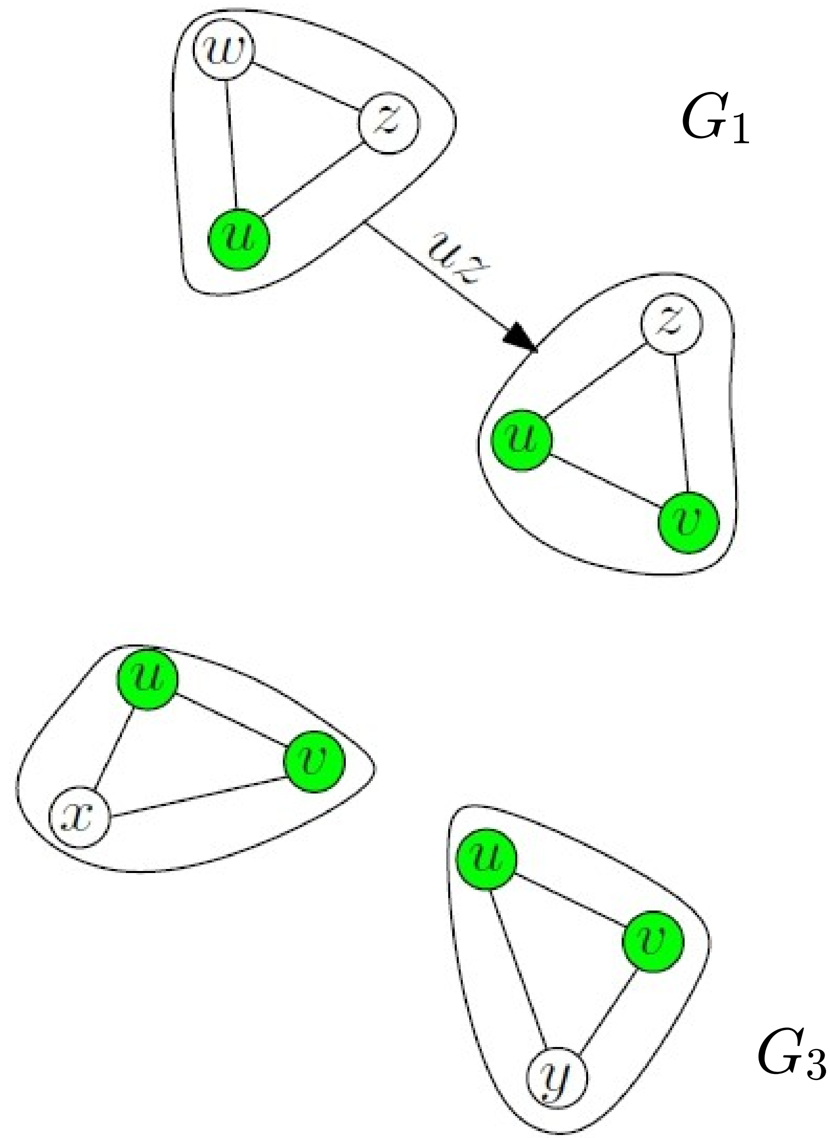

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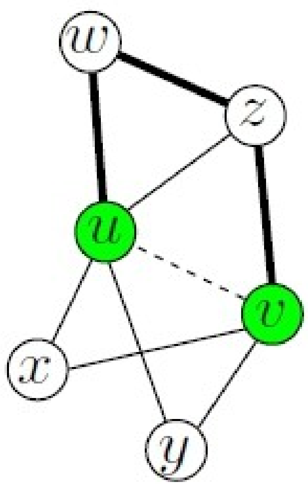

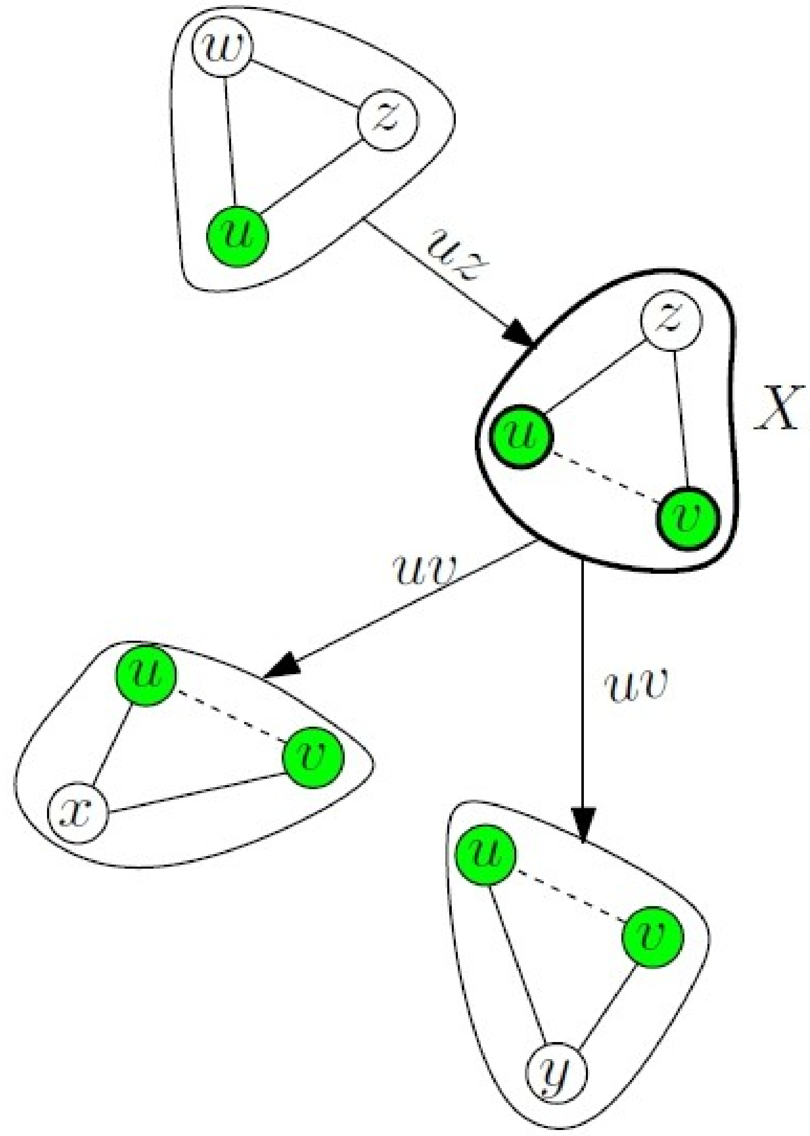

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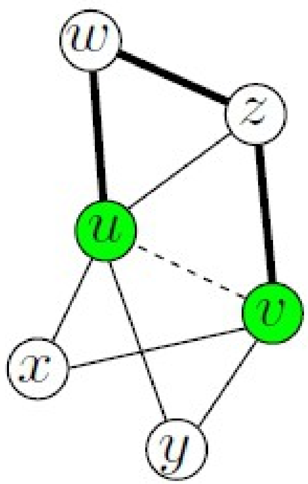

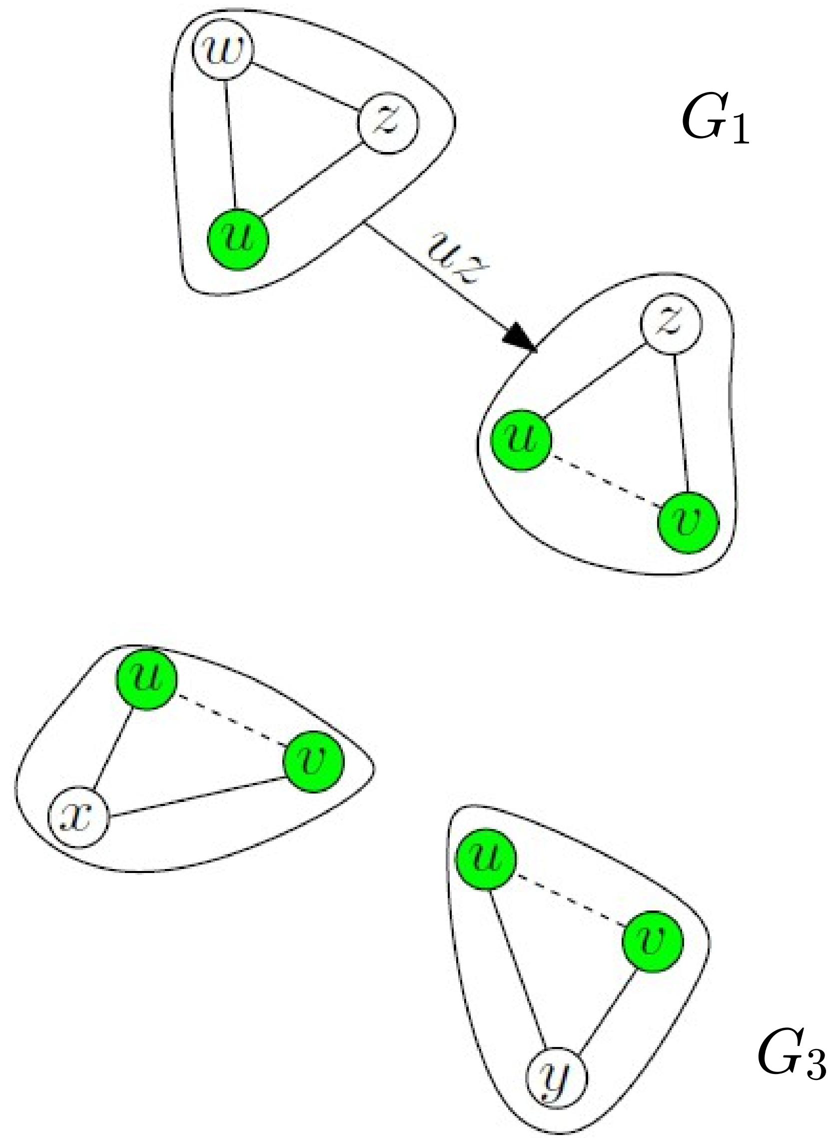

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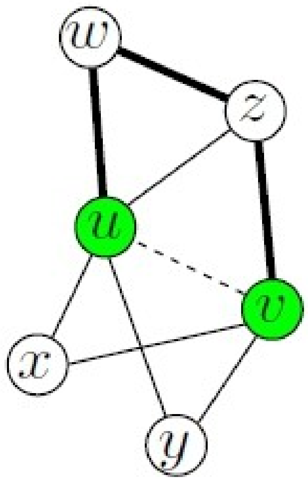
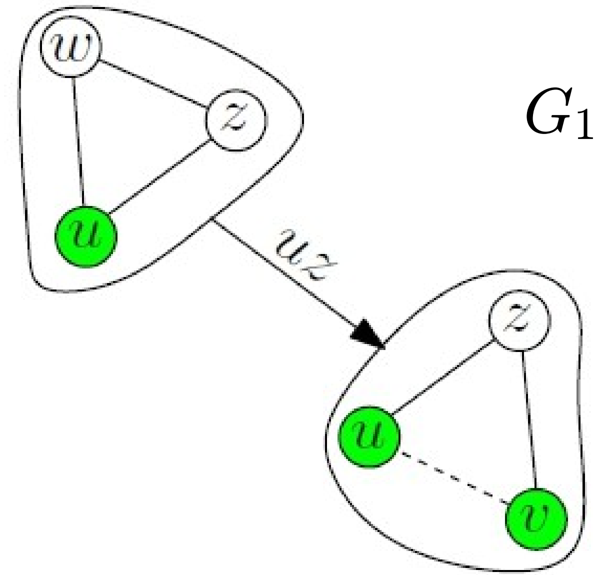
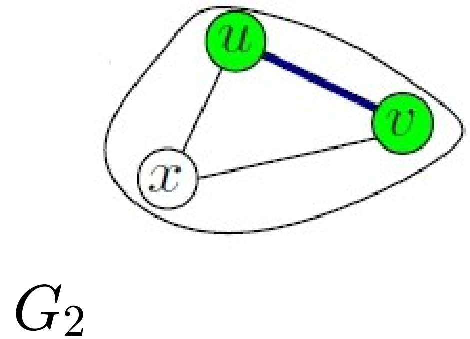
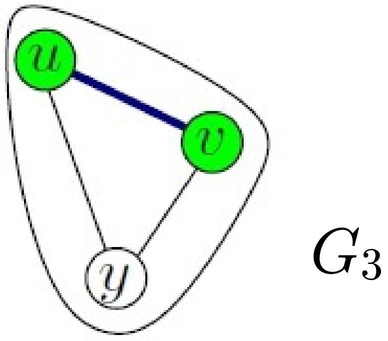




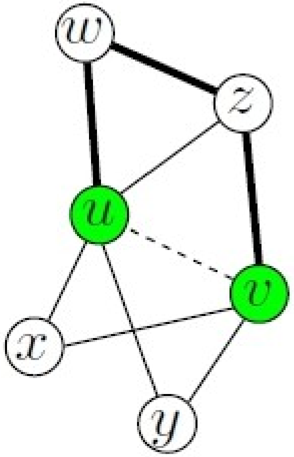




blue edges represent sp(u,v)

Algorithm

- Find all u,v such that G-u-v has at least 3 components



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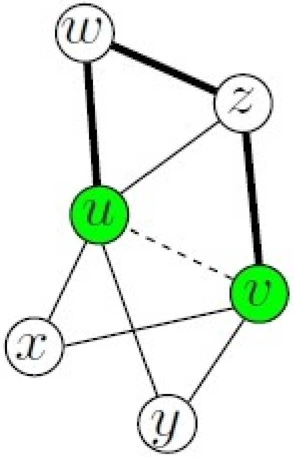

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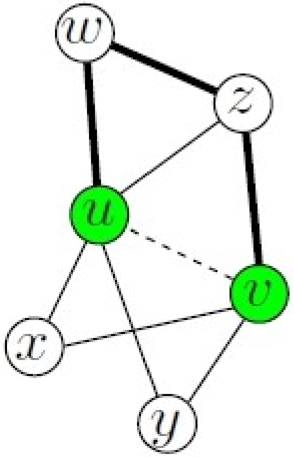

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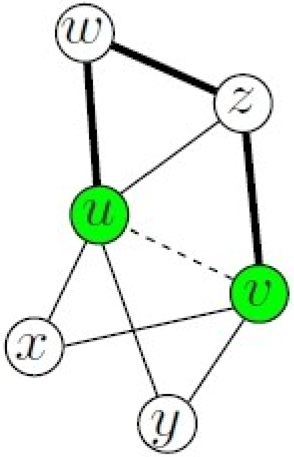

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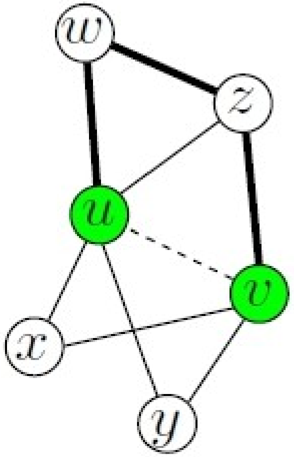

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- Compute MCB from $SC(G_i)$ and the SC from loose edges

Thank you!